Activity 29 Matrix algebra

Aim: Perform matrix calculations on ClassPad.

Two matrices are equal if and only if **every** corresponding element is equal, e.g.

if $\begin{bmatrix} 2 \\ a \end{bmatrix} = \begin{bmatrix} 2b \\ 3 \end{bmatrix}$ then $2 = 2b$ and $a = 3$.

1. Solve for a, b, c, d if

a) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} - \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

c) $2\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -1 & 9 \end{bmatrix}$

d) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(Hint: Multiply out the left-hand side and then solve simultaneously.)

e) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2. Rearrange to make **X** the subject and then solve for **X**, if

a) $\mathbf{X} + \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} -3 & 4 & 5 \\ 6 & -5 & 2 \end{bmatrix} + 3\mathbf{X} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \times \begin{bmatrix} 18 & -5 & 8 \end{bmatrix}$

3. The multiplicative inverse $\mathbf{A}^{-1}$ matrix has the property $\mathbf{AA}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$.

Determine the inverse of the matrix $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ by solving

$\begin{bmatrix} w & x \\ y & z \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ for w, x, y and z.

Enter and store the following matrices:

$$\mathbf{A} = \begin{bmatrix} 6 & -2 & 3 \\ 4 & 1 & -5 \end{bmatrix},\ \mathbf{B} = \begin{bmatrix} -1 & 3 \\ 2 & -2 \\ 5 & 1 \end{bmatrix},\ \mathbf{C} = \begin{bmatrix} 1 & 2 & -2 \\ 4 & 2 & -1 \\ 3 & -1 & 2 \end{bmatrix},\ \mathbf{D} = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

4. Calculate **X** in each equation using the matrices defined above.

- Make **X** the subject (showing your working)
- Evaluate **X**.

Example of working

$$\mathbf{XC} + 3\mathbf{E} = \begin{bmatrix} 5 & 1 & 7 \end{bmatrix}$$
$$\mathbf{XC} = \begin{bmatrix} 5 & 1 & 7 \end{bmatrix} - 3\mathbf{E}$$
$$\mathbf{XCC}^{-1} = \left(\begin{bmatrix} 5 & 1 & 7 \end{bmatrix} - 3\mathbf{E}\right)\mathbf{C}^{-1}$$
$$\mathbf{X} = \left(\begin{bmatrix} 5 & 1 & 7 \end{bmatrix} - 3\mathbf{E}\right)\mathbf{C}^{-1}$$
$$= \begin{bmatrix} -12 & 8 & -6 \end{bmatrix}$$

a) $3\mathbf{X} = \mathbf{B}$

b) $\mathbf{A} + \mathbf{X} = \begin{bmatrix} 2 & 0 & 2 \\ 7 & 1 & 0 \end{bmatrix}$

c) $\mathbf{CX} = \mathbf{BA}$

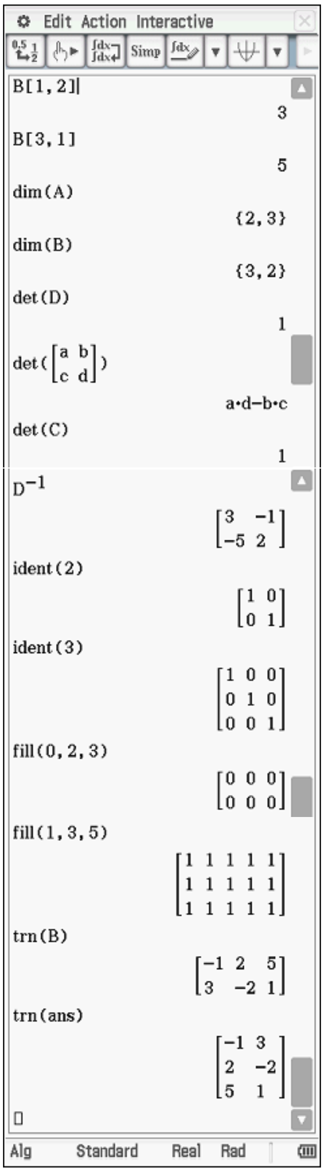
d) $\mathbf{AB} - 2\mathbf{X} = \mathbf{D}$

e) $\mathbf{XC} = \mathbf{BA}$

5. Each of the screenshots below involves a matrix function or calculation. For each:

   a) Duplicate in Main on your ClassPad.
      Note: matrices **A** to **E** are as used from the previous question.

   b) Edit the expressions to clarify the effect of each function and its arguments. Write a statement describing what each command does. Include an explanation of the function arguments. The first one has been done for you.



B[r,c] returns the element in the r^{th} row and c^{th} column of the matrix B.

dim

det

to the power –1

ident

fill

trn

Learning notes

How to enter matrices into ClassPad.

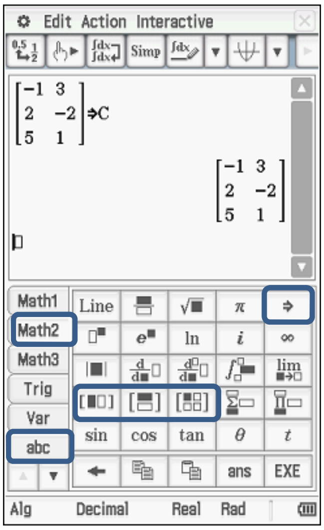
Enter and store a matrix, e.g. $\mathbf{A}=\begin{bmatrix}6 & -2 & 3\\ 4 & 1 & -5\end{bmatrix}$:

- Press [Keyboard], tap [Math2]

Use the matrix commands, Tap [⊞]

- Tap [▢]
- Enter the values using the arrow keys to navigate between elements
- Tap ⇒, Tap [abc]
- tap ⬆; tap A
- press [EXE]

Note: An alternative entry is [[6,–2,3][4,1,–5]]⇒A



Solving equations

There is no matrix division process. Equations of the form **AX=B** can be solved by pre-multiplying by $\mathbf{A}^{-1}$, the inverse of **A**.

For equations of the form, $\mathbf{XA}=\mathbf{B}$, post multiply by A^{-1}.

$$\begin{aligned}\mathbf{AX}&=\mathbf{B}\\ \mathbf{A}^{-1}\mathbf{AX}&=\mathbf{A}^{-1}\mathbf{B}\\ \mathbf{IX}&=\mathbf{A}^{-1}\mathbf{B}\\ \mathbf{X}&=\mathbf{A}^{-1}\mathbf{B}\end{aligned} \qquad \text{or} \qquad \begin{aligned}\mathbf{XA}&=\mathbf{B}\\ \mathbf{XAA}^{-1}&=\mathbf{BA}^{-1}\\ \mathbf{XI}&=\mathbf{BA}^{-1}\\ \mathbf{X}&=\mathbf{BA}^{-1}\end{aligned}$$

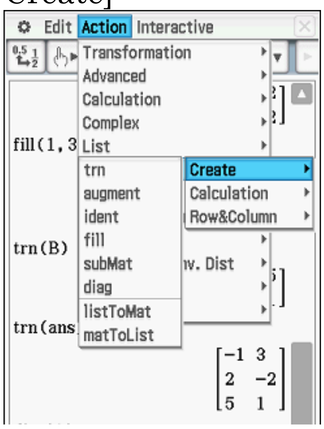
Multiplicative inverses of square matrices can be calculated when the determinant is non-zero. They are represented as $\mathbf{A}^{-1}$.

Access matrix functions from Main $\sqrt{\alpha}$ through

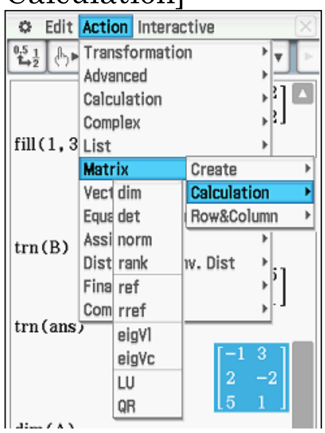
- [Action | Matrix-Create] or
- [Action | Matrix-Calculation].

To do Q5, it is useful to alter the function's arguments in order to see what each function does. Tap on the expression and change one of the numbers, repeat until the effect of changing that number is clear.

[Action | Matrix-Create]



[Action | Matrix-Calculation]



There are more matrix commands which you may choose to investigate.